Sequences Challenge Problems

1. Let $a_1 = 0$ and a_n to be the larger of a_{n-1} and $\sin n$ for all $n \geq 2$. Is a_n convergent? Why?

2. Let $a_{n+1} = \begin{cases} \frac{1}{2}a_n & \text{if } a_n \text{ is even} \\ 3a_n + 1 & \text{if } a_n \text{ is odd} \end{cases}$

   Compute the sequence with $a_1 = 34$ until you see a pattern. Do the same for $a_1 = 25$. What might you assume about this sequence? Can you prove it?

3. For each of the following, give an example or explain why none exist.

   (a) A bounded sequence that is not convergent.

   (b) A convergent sequence that is not bounded.

   (c) A monotone sequence that is not convergent.

   (d) A convergent sequence that is not monotone.

4. For each of the following, if the statement is true, explain why. If false, give a counterexample.

   (a) The sum of a convergent sequence and a divergent sequence diverges.

   (b) The sum of two divergent sequences diverges.

   (c) If $f(n) = a_n$ for all $n \in \mathbb{N}$ and $a_n \underset{n\to\infty}{\longrightarrow} L$, then $f(x) \underset{x\to\infty}{\longrightarrow} L$.

   (d) If $f(n) = a_n$ for all $n \in \mathbb{N}$ and $f(x) \underset{x\to\infty}{\longrightarrow} L$, then $a_n \underset{n\to\infty}{\longrightarrow} L$.