ODE Challenge Problems

1. Ignoring the constant of integration, find all functions f satisfying

$$\left(\int f(x)\,dx\right)\left(\int \frac{1}{f(x)}\,dx\right) = -1.$$

2. A dog sees a rabbit and chases it over an open field. Assume their movement satisfies the following:

   - The rabbit starts at the origin and the dog starts at the point $(L, 0)$.
   - The rabbit runs upward along the y-axis at a constant speed.
   - The dog runs straight at the rabbit at a constant speed.

   (a) Assuming that the dog runs at the same speed as the rabbit, show that the function representing the dog's path satisfies

   $$x\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

   .

   Find the solution to the ODE with the initial condition $y(L) = y'(L) = 0$. *Hint: Use $z = \frac{dy}{dx}$ to reduce the problem to a first order ODE, and then solve back for y.*

   Does the dog catch the rabbit?

   (b) Find another ODE and solve it assuming the dog travels twice as fast as the rabbit.

3. A pie is taken out of an oven at 5:00PM, and is at 100°C. At 5:10PM the pie is at 80°C, and at 5:20PM, is at 65°C. What is the temperature of the room? *Hint: Use **Newton's Law of Cooling:** $H' = k(H - H_R)$ where $H(t)$ is the temperture of the object, k is a proportionallity constant, and H_R is the room temperature, which is assumed constant.*