Math 6270 - Assignment 12

Due November 20, 2019

(1) Show that an equivalence between group extensions $N \to E_i \to G$ for $i = 1, 2$ yields an isomorphism between E_1, E_2.
(2) Give two group extensions of $\mathbb{Z}_3$ by $\mathbb{Z}_3$ that are isomorphic but not equivalent.
(3) See [1, page 316-317]. Let A be a G-module, $\phi \in Z^2(G, A)$. On $G \times A$ define

$$(x, a) \cdot (y, b) := (xy, ay + b + \phi(x, y))$$

for $x, y \in G, a, b \in A$. Here ay denotes the action of $y \in G$ on $a \in A$.
Check that $E(\phi) := (G \times A, \cdot)$ is a group with identity

$$(1, -\phi(1, 1))$$

and inverses

$$(x, a)^{-1} = (x, -ax^{-1} - \phi(1, 1) - \phi(x, x^{-1})).$$

[You do not have to type this up.]
(4) Continuing (3). Check that

$$0 \to A \xrightarrow{\mu} E(\phi) \xrightarrow{\varepsilon} G \to 1$$

is an extension with

$$\mu(a) := (1, a - \phi(1, 1)),$$
$$\varepsilon(x, a) := x.$$

Check that for the transversal function $\tau \colon G \to E(\phi),\ x \mapsto (x, 0)$, the extension induces the original G-module structure on A and the factor set ϕ.
[You do not have to type this up.]

References

[1] D. J. S. Robinson. *A course in the theory of groups*, volume 80 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1996.