Math 6270 - Assignment 11

Due November 13, 2019

All groups are assumed to be finite, representations and characters are over $\mathbb{C}$.

(1) Read pages 237-239 in [1] for a module theoretic approach to induced characters.
(2) Let H be a subgroup of G, and let φ a class function of H. Show that the induced function φ^G as defined in class is a class function on G.
(3) Let φ be a non-trivial character of A_4 of degree 1. Argue that the induced character φ^{A_5} is irreducible.
    Hint: This can be done without actually computing φ and φ^{A_5}. First find the degrees of all irreducible characters of A_5. Then use Frobenius reciprocity.
    Bonus: Give φ and φ^{A_5} explicitly.
(4) (a) Let G be a Frobenius group with kernel N. Show that $C_G(x) \leq N$ for all $x \in N \setminus \{1\}$.
    (b) Let G be a group with normal subgroup N such that $C_G(x) \leq N$ for all $x \in N \setminus \{1\}$. Show that N has a complement H in G. Is G a Frobenius group?

References

[1] D. J. S. Robinson. *A course in the theory of groups*, volume 80 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1996.