Math 6270 - Assignment 10

Due November 8, 2019

All groups are assumed to be finite, representations and characters are over $\mathbb{C}$.

(1) Show that $\mathrm{Irr}\,(G \times H) = \{\chi \times \sigma \mid \chi \in \mathrm{Irr}\,(G), \sigma \in \mathrm{Irr}\,(H)\}$.

(2) Let $\chi \in \mathrm{Irr}\,(G), \sigma \in \mathrm{Irr}\,(H)$ be faithful. Show that $\chi \times \sigma$ is faithful on $G \times H$ iff $\gcd(|Z(G)|, |Z(H)|) = 1$.

(3) Let φ be an irreducible representation of G. Show that $Z(\varphi(G))$ consists of scalar matrices.

(4) Prove the following generalization of Burnside's $p^a q^b$-Theorem: a finite group with nilpotent subgroup of prime power index is solvable.

Hint: consider a central element in the nilpotent subgroup.