Math 6270 - Assignment 9

Due October 30, 2019

All groups are assumed to be finite, representations and characters are over $\mathbb{C}$.

(1) Let φ be a $\mathbb{C}$-representation of G with character χ and let $g \in G$. Show
  (a) $\chi(g) = \chi(1)$ iff $g \in \ker \varphi$.
  (b) $|\chi(g)| \leq \chi(1)$ with equality iff $\varphi(g)$ is a multiple of the identity matrix.
  (c) $\chi(g^{-1}) = \overline{\chi(g)}$.
  Hint: Decompose the restriction of φ to $\langle g \rangle$ into irreducible representations.

(2) Let $g \in G$. Show that g is conjugate to g^{-1} in G iff $\chi(g)$ is real for all characters χ of G.

(3) The **kernel** of a character χ of G is defined as $\ker \chi := \{g \in G \mid \chi(g) = \chi(1)\}$.
  Show for any $N \trianglelefteq G$ that

$$N = \bigcap \{\ker \chi \mid \chi \in \operatorname{Irr}(G), N \leq \ker \chi\}$$

(4) Read the example on page 233 of [1] for computing the character table of the quaternion group Q_8. Argue that the character table of D_8 is the same.

References

[1] D. J. S. Robinson. *A course in the theory of groups*, volume 80 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1996.