

Math 6270 - Assignment 8

Due October 23, 2019

- (1) Show the converse of Maschke's Theorem: Let G be a finite group whose order is a multiple of the characteristic of the field F . Then FG is not completely reducible as regular FG -module.

Hint: Show that the submodule $A := \{\sum_{g \in G} c_g g \mid \sum_{g \in G} c_g = 0\}$ of FG intersects every nontrivial submodule nontrivially.

- (2) Let $n \in \mathbb{N}$ and F a field.
- (a) Determine the minimal right ideals of $F^{n \times n}$.
 - (b) Show that $F^{n \times n}$ is a simple ring (i.e., has no proper non-trivial ideals).
- (3) Let G be a cyclic group of order n . Determine all irreducible representations of G over \mathbb{C}, \mathbb{Q} , respectively. Which are faithful?
Describe the structure of $\mathbb{C}G$ and of $\mathbb{Q}G$.
- (4) Describe the structure of $\mathbb{C}A_4$. Find all irreducible \mathbb{C} -representations of A_4 .