Math 6270 - Assignment 7

Due October 16, 2019

(1) Read 10.1.9 and the remark afterwards in [1].

(2) Let G be a finite group, and let p be a prime divisor of $|G' \cap Z(G)|$. Show that no Sylow p-subgroup P of G is abelian.

Hint: Seeking a contradiction, suppose P is abelian and consider the transfer τ from G into P. Pick a non-identity element $z \in P \cap G' \cap Z(G)$ and show that $\tau(z) = z^{|G:P|}$. Consider the kernel of τ to get a contradiction.

(3) Show that every group of order n is cyclic iff $\gcd(n, \varphi(n)) = 1$.

(4) Let a group G act on $\{1, \ldots, n\}$ and let V be a vector space with basis $b_1, \ldots, b_n$. For $g \in G$ define an automorphism $\varphi(g)$ of V by

$$\varphi(g)\colon\, b_x \mapsto b_{x \bullet g} \text{ for all } 1 \leq x \leq n.$$

Then $\varphi\colon G \to \operatorname{Aut}(V)$ is called a (linear) **permutation representation** of G.

Show that φ is reducible if $n > 1$.

References

[1] D. J. S. Robinson. *A course in the theory of groups*, volume 80 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1996.