Math 6270 - Assignment 5

Due September 25, 2019

(1) Read pages 73-78 in [1] on the simplicity of projective special linear groups.
(2) Let G be a p-group for a prime p. Show that $\mathrm{Frat}(G) = G'G^p$.
(3) A group G is **supersolvable** if it has a normal series

$$1 = N_0 \leq N_1 \leq \cdots \leq N_\ell = G$$

where $N_i \trianglelefteq G$ and N_{i+1}/N_i is cyclic for all $i < \ell$.

Clearly, nilpotent $\Rightarrow$ supersolvable $\Rightarrow$ solvable.

Show for G finite and supersolvable:

(a) Every minimal normal subgroup of G has prime order.
(b) The index of every maximal subgroup of G is prime.
Hint: Use a minimal normal subgroup and induction.

(4) Let G be a finite solvable group with a minimal normal subgroup N such that $C_G(N) = N$. Show that N has a complement in G and all such complements are conjugate.

Hint: Let L/N be minimal normal in G/N. Show that N has a complement H in L and that $N_G(H)$ is a complement of N in G.

References

[1] D. J. S. Robinson. *A course in the theory of groups*, volume 80 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1996.