Math 6270 - Assignment 2

Due September 11, 2019

(1) Read Section 2.1 on free groups (in particular on reduced words, normal forms, concrete examples of free groups and the projective property) in [1].

(2) Let F_1, F_2 be free over X_1, X_2, respectively, and $F_1 \cong F_2$. Show that $|X_1| = |X_2|$.

Hint: Consider the set of homomorphisms from F_i to $\mathbb{Z}_2$ as vector spaces over $\mathbb{Z}_2$.

(3) Show that every group that has a finite presentation with strictly more generators than relations is infinite.

Hint: Let A be an abelian group with generators $x_1, \ldots, x_k$ and defining relations $[x_i, x_j] = 1$ for all $1 \leq i < j \leq k$ and r additional relations. Show that A is infinite if $r < k$.

(4) Show that $G = \langle x, y : x^m = y^n = 1 \rangle$ is infinite for $m, n > 1$.

References

[1] D. J. S. Robinson. *A course in the theory of groups*, volume 80 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1996.