Math 6010 - Assignment 3

Due February 10, 2019

(1) Prove or disprove:
   (a) For computably enumerable languages A, B, also AB is c.e.
   (b) For A, B c.e., also $A \setminus B$ is c.e.

(2) Complete the proof from class:
   (a) A partial function $f\colon \Sigma^* \to_p \Sigma^*$ is computable iff its graph
   $$L_f = \{(x, y) \in \Sigma^* \times \Sigma^* \mid x \in \text{domain}(f), f(x) = y\}$$
   is computably enumerable.
   (b) If a total function is computable, then its graph is computable.

(3) Argue that the language of multivariate polynomials (in some appropriate coding) over the integers that have an integer root is computably enumerable.

(4) Show that
   $$\text{succ} := \lambda nfx.f((nf)x)$$
   defines the successor function on natural numbers (Church numerals) in λ-calculus.