

Math 6010 - Assignment 10

Due April 29, 2019

- (1) Graphs $G = (V, E)$ and $H = (W, F)$ are isomorphic if there exists a bijection $f: V \rightarrow W$ such that for all $i, j \in V$: $(i, j) \in E$ iff $(f(i), f(j)) \in F$.

Show that

$\text{GraphIsomorphism} := \{(G, H) \mid G, H \text{ are isomorphic graphs}\}$

is in NP.

- (2) Show that

$\text{Primes} := \{n \mid n \text{ is a prime in binary}\}$

is in NP.

Use the following fact: For $n > 1$, the multiplicative group $\mathbb{Z}_n^* := \{x \in \mathbb{Z}_n \mid x \text{ is a unit}\}$ is cyclic of order $n - 1$ iff n is prime. Then obtain a witness for primality of n from the prime factors of $n - 1$.

- (3) Show $\text{NL} \neq \text{PSPACE}$.
(4) Show $\text{PSPACE} \neq \text{EXPSPACE}$ and $\text{P} \neq \text{EXPTIME}$.