Math 6010 - Assignment 9

<div align="center">Due April 10, 2019</div>

(1) Show that for every $f \colon \mathbb{N} \to \mathbb{N}$ there exists $A \subseteq \mathbb{N}$ such that $\deg_T(f) = \deg_T(\chi_A)$.

Hint: Encode the graph of f into a subset of $\mathbb{N}$.

(2) Prove the

Complementation Theorem. Let $A, B \subseteq \mathbb{N}$. Then $A \leq_T B$ iff A and $\bar{A}$ are recursively enumerable in B.

(3) Let $A \subseteq \mathbb{N}$. Show
$$A <_T A'$$
(i.e. A is Turing reducible to its jump $A' = \{x \mid \varphi_x^A(x) \downarrow\}$ but not conversely).

(4) Let $A, B \subseteq \mathbb{N}$. Prove
$$A \leq_T B \text{ iff } A' \leq_m B'.$$

Hint: For the "if"-direction show that A and $\bar{A}$ are B-recursively enumerable using the Relativized Parameter and Enumeration Theorems.