Math 6010 - Assignment 7

Due March 8, 2019

(1) Show that Σ_n^0 for $n \in \mathbb{N}$ is closed under $\wedge, \vee$ and bounded quantifiers.

(2) Prove that the following are equivalent for $A \subseteq \mathbb{N}$:
   (a) A is recursively enumerable.
   (b) A is the domain of some partial recursive function.
   (c) A is in Σ_1^0.
   (d) A is the range of a partial recursive function $f\colon \mathbb{N} \to \mathbb{N}$.

(3) Prove that a partial function is recursive iff its graph is recursively enumerable.

(4) For $n \in \mathbb{N}$ let $\Delta_n^0 := \Sigma_n^0 \cap \Pi_n^0$. Show that

$$\Sigma_n^0 \cup \Pi_n^0 \subsetneq \Delta_{n+1}^0$$

for $n \geq 1$. What about $n = 0$?

Hint: To show that the containment is proper let P be Σ_n^0 but not Π_n^0 and consider

$$Q(x, z) := (P(x) \wedge z = 0) \vee (\neg P(x) \wedge z = 1).$$