

# Math 6010 - Assignment 10

Due November 18, 2015

- (36) Prove that for every  $f(n)$ -time non-deterministic Turing machine there exist a constant  $c$  and an equivalent  $O(c^{f(n)})$ -time deterministic Turing machine.
- (37) Graphs  $G = (V, E)$  and  $H = (W, F)$  are isomorphic if there exists a bijection  $f: V \rightarrow W$  such that for all  $i, j \in V$ :  $(i, j) \in E$  iff  $(f(i), f(j)) \in F$ .

Show that

$\text{GraphIsomorphism} := \{(G, H) \mid G, H \text{ are isomorphic graphs}\}$

is in NP.

- (38) Show that

$\text{Primes} := \{n \mid n \text{ is a prime in binary}\}$

is in NP.

Use the following fact: For  $n > 1$ , the multiplicative group  $\mathbb{Z}_n^* := \{x \in \mathbb{Z}_n \mid x \text{ is a unit}\}$  is cyclic of order  $n - 1$  iff  $n$  is prime. Then obtain a witness for primality of  $n$  from the prime factors of  $n - 1$ .