

# Math 6010 - Assignment 9

Due October 28, 2015

- (33) Show that every Turing degree contains (the characteristic function of) a set.

In particular, the poset of degrees of functions is the poset of degrees of subsets of  $\mathbb{N}$ .

Hint: Code the graph of a given function  $f: \mathbb{N} \rightarrow \mathbb{N}$  as a subset of  $\mathbb{N}$ .

- (34) Let  $A \subseteq \mathbb{N}$ . Show

$$A <_T A'$$

(i.e.  $A$  is Turing reducible to its jump  $A' = \{x \mid \varphi_x^A(x) \downarrow\}$  but not conversely).

- (35) Let  $A, B \subseteq \mathbb{N}$ . Prove

$$A \leq_T B \text{ iff } A' \leq_m B'.$$

Hint: For the “if”-direction show that  $A$  and  $\bar{A}$  are  $B$ -recursively enumerable using the Relativized Parameter and Enumeration Theorems.