# Math 6010 - Assignment 8

Due October 21, 2015

Recall that a sequence of natural numbers $a_0, \dots, a_n$ can be uniquely encoded as natural number $\langle a_0, \dots, a_n \rangle := \prod_{i=0}^{n} p_i^{1+a_i}$.

(32) [1, Exercise I 3.11] We give an alternative definition of $\varphi_{e,s}(x)$ by recursion on $s$:

Let $\varphi_{e,0}(x) \uparrow$. For $s \in \mathbb{N}$ let $\varphi_{e,s+1}(x) := y$ if $\varphi_{e,s}(x) = y$ or if $s = \langle e, x, y, t \rangle$ for some $t > 0$ and $y$ is the output $\varphi_e(x)$ of the Turing machine $M_e$ in $\leqslant t$ steps.

Further $W_{e,s} := \operatorname{dom} \varphi_{e,s}$.

Check the following:

(a) $\{(e, s, x) \in \mathbb{N}^3 \mid \varphi_{e,s}(x) \downarrow\}$ is recursive.

(b) $\{(e, s, x, y) \in \mathbb{N}^4 \mid \varphi_{e,s}(x) = y\}$ is recursive.

(c) $\varphi_{e,s}(x) = y \Rightarrow e, x, y < s$.

(d) For any $s \in \mathbb{N}$ there exists at most one pair $(e, x) \in \mathbb{N}^2$ such that $x \in W_{e,s+1} \backslash W_{e,s}$.

## References


[1] Soare, Robert I. Recursively enumerable sets and degrees. Springer-Verlag, Berlin, 1987.