Math 3140 - Assignment 13

Due December 3, 2021

(1) Show every finite abelian group is the direct product of its Sylow subgroups.

(2) Let $n \in \mathbb{N}$ be odd.
  (a) Give a Sylow 2-subgroup of D_{2n}. What is it isomorphic to? How many are there?
  (b) Let p an odd prime. What are the Sylow p-subgroups of D_{2n}?
  (c) Are any of the Sylow subgroups of D_{2n} normal?

(3) For every prime p give a Sylow p-subgroup of A_5. Can you determine how many there are for each p? Are any of them normal?

  Hint: Look at Exercises 11.5-6 for the number of permutations of a certain cycle structure.

(4) How many groups of size 21 are there up to isomorphism? What do they look like?

  How many groups of size 33? (cf. Example 5.4.12 of [1]).

References

[1] Frederick M. Goodman. Algebra: abstract and concrete. SemiSimple Press, edition 2.6, 2015. Available from `http://www.math.uiowa.edu/~goodman`