Math 3140 - Assignment 9

Due October 27, 2021

(1) Are the following groups isomorphic? For (a), (b) do not use the Fundamental Theorem of Finite Abelian Groups but count the elements of fixed order.
  (a) $\mathbb{Z}_8$ and $\mathbb{Z}_4 \times \mathbb{Z}_2$
  (b) $\mathbb{Z}_4 \times \mathbb{Z}_4$ and $\mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$
  (c) $\mathbb{Z}_{10} \times \mathbb{Z}_{12} \times \mathbb{Z}_6$ and $\mathbb{Z}_{60} \times \mathbb{Z}_6 \times \mathbb{Z}_2$
  (d) $\mathbb{Z}_{10} \times \mathbb{Z}_{12} \times \mathbb{Z}_6$ and $\mathbb{Z}_{15} \times \mathbb{Z}_4 \times \mathbb{Z}_{12}$

(2) How many abelian groups up to isomorphism are there of order
  (a) 6,
  (b) 15,
  (c) 30,
  (d) pq for distinct primes p, q
  (e) n where n is a product of pairwise distinct primes?

(3) Find all abelian groups of order 180 up to isomorphism.

(4) How many abelian groups are there of order 3^5 up to isomorphism? How many of order p^5 where p is prime?

(5) The *exponent* of a group G is the smallest positive integer n such that $x^n = 1$ for all $x \in G$. What is the exponent of
  (a) S_3,
  (b) D_8,
  (c) $\mathbb{Z}_9 \times Z_3$?

  How many abelian groups up to isomorphism are there of order 16 and exponent 4?

(6) Let $(A, +)$ be an abelian group. The set

$$A_{\text{tor}} := \{x \in A : x \text{ has finite order}\}$$

  of elements of finite order is called the *torsion part* of A.

  Show that A_{tor} is a subgroup of $(A, +)$.

(7) Let $A = \mathbb{Z}_{n_1} \times \cdots \times \mathbb{Z}_{n_k} \times \mathbb{Z}^r$.
  (a) What is the torsion part of A?
  (b) What is A/A_{tor} isomorphic to?