Math 3140 - Assignment 8

Due October 20, 2021

(1) Show that for every group G conjugacy of elements is an equivalence relation.

Its equivalence classes are called the **conjugacy classes** of G.

(2) (a) Show that a subgroup N of G is normal iff N is a union of conjugacy classes of G.
  (b) Which conjugacy classes are contained in the center $Z(G)$?

(3) Use Exercise 7.4 and the previous to determine all normal subgroups of S_3.
  (a) What is $Z(S_3)$?
  (b) Describe the quotient groups S_3/N for these normal subgroups up to isomorphism as simple as possible.
  (c) Is there a surjective homomorphism from S_3 onto $\mathbb{Z}_3$?

(4) Complete the proof of the **Homomorphism Theorem** from class.

(5) Prove

$$G \times H \ / \ G \times \{1_H\} \cong H.$$

Hint: Use the Homomorphism Theorem for an appropriately chosen homomorphism.

(6) **Correspondence Theorem between normal subgroups:** Let $\varphi\colon G \to H$ be an onto homomorphism. Show
  (a) If B is normal in H, then $\varphi^{-1}(B)$ is normal in G.
  (b) If A is normal in G, then $\varphi(A)$ is normal in H.

(7) Let $n \in \mathbb{N}$. Show that every subgroup of $\mathbb{Z}_n$ is generated by some $[d]$ where d is an integer that divides n.
Hint: Use the Correspondence Theorem and the description of subgroups of $\mathbb{Z}$ from class.