Math 3140 - Assignment 6

Due October 6, 2021

(1) For the following subgroups H of G, find all the left cosets of H in G. Give one representative for each left coset. How many are there?
  (a) $G = \mathbb{R}^2$ under addition, $H = \{(x, 0) \ : \ x \in \mathbb{R}\}$
  (b) $G = \langle a \rangle$ of order 12, $H = \langle a^4 \rangle$
  (c) $G = \mathbb{R}^*$ under multiplication, $H = \mathbb{R}^+$ the subgroup of positive reals

(2) Show that if the order of a group G is a prime p, then it is isomorphic to $(\mathbb{Z}_p, +)$.

(3) Let G be a nontrivial group that has no proper, nontrivial subgroups (i.e. 1 and G are the only subgroups of G). Show that $|G|$ is prime.

    Hint: Do not assume at the outset that G is finite.

(4) For any integer $n > 1$, Euler's ϕ-function $\phi(n)$ yields the number of positive integers less than n that are coprime to n. Prove:

    Euler's Theorem. If a is coprime to n, then $a^{\phi(n)} \equiv 1 \bmod n$.

(5) Let G be a finite group with subgroups $H \leq K \leq G$. Show that

$$|G : H| = |G : K| \cdot |K : H|.$$