Math 3140 - Assignment 5

Due September 27, 2021

Do the following as practice exam for midterm 1 on September 29.

(1) Prove or disprove:
$\mathbb{Z}$ with the operation $x \oplus y := x + y + 3$ for $x, y \in \mathbb{Z}$ is a group.

(2) Let $(G, \cdot)$ be a group and $g \in G$. Show that

$$a_g \colon G \to G,\ x \mapsto gx,$$

is a permutation on G.
Is a_g a homomorphism on G?

(3) Let $(G, \cdot)$ be a group and $S \subseteq G$. Show that the subgroup generated by S consists of all finite products of integer powers of elements in S, i.e.

$$\langle S \rangle = \{a_1^{k_1} \cdots a_n^{k_n} \ :\ n \geq 0, k_1, \ldots, k_n \in \mathbb{Z}, a_1, \ldots, a_n \in S\}$$

(4) Let $\varphi \colon G \to H$ be a group isomorphism, let $a \in G$.
Show that the order of a in G is equal to the order of $\varphi(a)$ in H.

(5) Show that $\varphi \colon G \to G,\ x \mapsto x^{-1}$, is a homomorphism iff $(G, \cdot)$ is commutative (i.e., $xy = yx$ for all $x, y \in G$).