Math 3140 - Assignment 4

Due September 15, 2021

(1) Which of the following are subgroups of S_4?
   (a) $A = \{(), (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\}$
   (b) $B = \{f \in S_4 : f(2) > f(1)\}$
(2) Let A, B be subgroups of a group $(G, \cdot)$. Show that $A \cap B$ is a subgroup as well.
(3) Determine the center of $\mathrm{GL}(2, \mathbb{R})$.

   Hint: Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in Z(\mathrm{GL}(2, \mathbb{R}))$. Consider its products with matrices E_{ij} whose ij-entry is 1 and all other entries are 0. These are not invertible but their sum with the identity matrix $I + E_{ij}$ is.
(4) Which of the following groups are cyclic? For those that are, list all their generators. For those that are not, explain why.

   $A = (\mathbb{Q}, +)$
   $B = (\mathbb{Z}_{12}, +)$
   $C = (\mathbb{Z}_7^*, \cdot)$
   $D = \{\pi^z \ : \ z \in \mathbb{Z}\}$ under multiplication
   $E = \mathbb{Z}^2 = \{(a, b) \ : \ a, b \in \mathbb{Z}\}$ under addition