Math 3130 - Assignment 1

Due September 1, 2021

(1) Complete the multiplication table for the symmetries $1, a, b, c$ of a (nonsquare) rectangle.

A multiplication is *commutative* if order of the arguments does not matter, that is, $xy = yx$ for all x and y.

Is the multiplication of symmetries of a rectangle commutative?

(2) Continue the example from the lecture and find the remaining standard matrices for the symmetries $1, r, r^2, a, b, c$ of an equilateral triangle with vertices at $(1, 0)$, $(-1/2, \sqrt{3}/2)$, $(-1/2, -\sqrt{3}/2)$.

Here r is the rotation by $2\pi/3$ counterclockwise around the origin, a the reflection at the x-axis, b the reflection at the axis through $(-1/2, \sqrt{3}/2)$, and c the reflection at the axis through $(-1/2, -\sqrt{3}/2)$.

(3) Represent the symmetries of an equilateral triangle with vertices labelled $1, 2, 3$ as permutations.

(Use the same notation as in the previous problem with vertices $(1, 0)$, $(-1/2, \sqrt{3}/2)$, $(-1/2, -\sqrt{3}/2)$ labelled $1, 2, 3$, respectively).

(a) What are the inverses for each symmetry?

(b) Is the composition of symmetries of an equilateral triangle commutative?

(4) A *regular n-gon* is a plane figure that has n sides of equal length and n equal angles. E.g., for $n = 4$ a regular n-gon is just a square.

(a) What is the number of symmetries of a regular pentagon (5-gon?

Hint: Make a sketch, label the vertices and count the options where they might be mapped by a symmetry.

(b) How many symmetries are there for a regular n-gon ($n \geq 3$)? Describe them in words.

The group of symmetries of a regular n-gon is called the *dihedral group of order* $2n$ and denoted D_{2n} for short.

(5) Write the following permutations in cycle notation:

$$a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 1 & 6 & 5 & 7 & 2 \end{pmatrix} \qquad b = (1\ 2)(1\ 2\ 3\ 4)$$

$$c = (1\ 5)(1\ 4)(1\ 3)(1\ 2) \qquad d = [(1\ 2\ 3\ 4)(5\ 6\ 7)]^{-1}$$