Math 3140 - Assignment 13

Due November 30, 2016

(1) Assume that $G/Z(G)$ is cyclic. Show that G is abelian.

Hint: There exists $a \in G$ such that every element in G is of the form $a^i z$ for some $z \in Z(G)$ and $i \in \mathbb{Z}$. Why?

(2) Show every finite abelian group is the direct product of its Sylow subgroups.

(3) Let $n \in \mathbb{N}$ be odd.
   (a) Give a Sylow 2-subgroup of D_{2n}. What is it isomorphic to? How many are there?
   (b) Let p an odd prime. What are the Sylow p-subgroups of D_{2n}?
   (c) Are any of the Sylow subgroups of D_{2n} normal?

(4) For every prime p give a Sylow p-subgroup of A_5. Can you determine how many there are for each p? Are any of them normal?

Hint: Look at Exercise 12.5 for the number of permutations of a certain cycle structure.

(5) How many groups of size 21 are there up to isomorphism? What do they look like?

How many groups of size 33? (cf. Example 5.4.12 of [1]).

References

[1] Frederick M. Goodman. Algebra: abstract and concrete. SemiSimple Press, edition 2.6, 2015. Available from `http://www.math.uiowa.edu/~goodman`