Math 3140 - Assignment 12

Due November 16, 2016

(1) Read pages 220-221 in [1] on the rotational symmetries of the cube:
   (a) How many are there?
   (b) Number the faces of the cube $1, \dots, 6$. What are the cycle structures for the rotational symmetries on $1, \dots, 6$? How often does each cycle structure appear in this group?

(2) How many colorings are there of the faces of a cube in 2 colors up to rotational symmetry? (Two colorings are considered equivalent when one can be obtained from the other by rotating the cube.)

   Hint: exercise (1)

(3) (a) For $g = (1\ 2\ 3\ 4)(5\ 6)$ determine $C_{S_6}(g)$, the centralizer of g in S_6.
   (b) How many elements in S_6 are conjugate to $(1\ 2\ 3\ 4)(5\ 6)$?

(4) How many elements in S_6 are a product of 2 disjoint cycles of length 3 each?

   Hint: What is the size of the centralizer of such an element?

(5) Assume that a permutation g in S_n has a decomposition into m_1 cycles of length k_1, m_2 cycles of length k_2, ..., and m_ℓ cycles of length k_ℓ where $k_1 > k_2 > \dots > k_\ell$ and $m_1, \dots, m_\ell$ are positive integers.

   Determine the size of the conjugacy class of g in S_n.

References

[1] Frederick M. Goodman. Algebra: abstract and concrete. SemiSimple Press, edition 2.6, 2015. Available from `http://www.math.uiowa.edu/~goodman`