Math 3140 - Assignment 10

Due November 2, 2016

(1) Find all abelian groups of order 180 up to isomorphism.
(2) How many abelian groups are there of order 3^5 up to isomorphism? How many of order p^5 where p is prime?
(3) Let $(A, +)$ be an abelian group, $a_1, \ldots, a_n \in A$. Show that

$$B = \{z_1 a_1 + \cdots + z_n a_n \ : \ z_1, \ldots, z_n \in \mathbb{Z}\}$$

is a subgroup of $(A, +)$.
(4) Let $(A, +)$ be an abelian group. The set

$$A_{\text{tor}} := \{x \in A \ : \ o(x) \text{ is finite}\}$$

of elements of finite order is called the *torsion part* of A.
Show that A_{tor} is a subgroup of $(A, +)$.
(5) Let $A = \mathbb{Z}_{n_1} \times \cdots \times \mathbb{Z}_{n_k} \times \mathbb{Z}^r$.
   (a) What is the torsion part of A?
   (b) What is A/A_{tor} isomorphic to?