Math 3140 - Assignment 8

Due October 19, 2016

(1) Let N be a normal subgroup of G such that G/N is abelian. Show that $x^{-1}y^{-1}xy \in N$ for all $x, y \in G$.

(The expression $x^{-1}y^{-1}xy$ is called the *commutator* of x and y and denoted by $[x, y]$.)

(2) Let G' be the normal subgroup of G that is generated by the set of all commutators $\{[x, y] : x, y \in G\}$. Show that G/G' is abelian.

(G' is called the *commutator subgroup* or *derived subgroup* of G)

(3) Let $D_8 = \langle a, b\rangle$ be the symmetry group of the square.
   (a) Show that the derived subgroup D_8' is $\langle a^2\rangle$.
   (b) Which group is D_8/D_8' isomorphic to?

(4) Correspondence Theorem between normal subgroups: Let $\varphi\colon G \to H$ be an onto homomorphism. Show
   (a) If B is normal in H, then $\varphi^{-1}(B)$ is normal in G.
   (b) If A is normal in G, then $\varphi(A)$ is normal in H.

(5) Let $n \in \mathbb{N}$. Show that every subgroup of $\mathbb{Z}_n$ is generated by some $[d]$ where d is an integer that divides n. Use the Correspondence Theorem and the description of subgroups of $\mathbb{Z}$.

(6) Let $m, n \in \mathbb{N}$ be relatively prime. Show that $\mathbb{Z}_m \times \mathbb{Z}_n$ is isomorphic to $\mathbb{Z}_{mn}$.