Math 3140 - Assignment 5

Due September 28, 2016

(1) Find a subgroup H of S_4 with $|H| = 4$ and H not isomorphic to $(\mathbb{Z}_4, +)$.

Hint: Consider the symmetries of a rectangle. Why are they not isomorphic to $(\mathbb{Z}_4, +)$?

(2) Let G be a group, and $a \in G$ of finite order, i.e. $o(a) = |\langle a \rangle|$ is finite. Show that $o(a)$ is the smallest $n \in \mathbb{N}$ such that $a^n = 1$.

Hint: Show for this n that $1, a, a^2, \ldots, a^{n-1}$ are all distinct.

(3) Let $\varphi\colon G \to H$ be a group homomorphism and $B \leq H$ a subgroup of H. Show that the preimage

$$\varphi^{-1}(B) := \{g \in G\colon \varphi(g) \in B\}$$

is a subgroup of G.

(4) For $n \in \mathbb{N}$, let $(\mathbb{Z}_n^*, \cdot)$ denote the group of invertible elements in $\mathbb{Z}_n$ with multiplication.

(a) Show that $\mathbb{Z}_7^*$ is cyclic. Which elements are generators?

(b) What are the orders of the elements in $\mathbb{Z}_7^*$?

(c) Which other group is $(\mathbb{Z}_7^*, \cdot)$ isomorphic to? Give the isomorphism.