Math 3140 - Assignment 4

Due September 21, 2016

Problems 1-3a are reviewing material for the 1st midterm.

(1) Compute the multiplicative inverses of the following if they exist:
   (a) $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ in $\mathrm{GL}(2, \mathbb{R})$
   (b) $b = (2\ 3\ 4)(1\ 2\ 3)$ in S_4 (give a decomposition in disjoint cycles)
   (c) $c = [9]$ in $\mathbb{Z}_{25}$

(2) [1, Exercise 1.5.6] A permutation that is a cycle of length 2 is called a *transposition*.
   Show that any cycle $(a_1\ a_2\ \ldots a_k)$ of length k can be written as a product of transpositions. Conclude that any permutation is a product of transpositions.
   Hint: For the first part look at problem 5c of assignment 1 to discover the right pattern.

(3) Which of the following are subgroups of S_4?
   (a) $A = \{(), (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\}$
   (b) $B = \{f \in S_4 \colon f(2) > f(1)\}$

(4) Let A, B be subgroups of a group $(G, \cdot)$. Show that $A \cap B$ is a subgroup as well.

(5) Let $\varphi\colon G \to H$ be a homomorphism between the group $(G, \cdot)$ with identity 1_G and the group $(H, *)$ with identity 1_H. Show:
   (a) $\varphi(1_G) = 1_H$
   Hint: Start by evaluating $\varphi(1_G \cdot 1_G)$ in two ways.
   (b) $\varphi(g^{-1}) = \varphi(g)^{-1}$ for all $g \in G$.
   Hint: Use (a).

References

[1] Frederick M. Goodman. Algebra: abstract and concrete. SemiSimple Press, edition 2.6, 2015. Available from `http://www.math.uiowa.edu/~goodman`