Math 3130 - Assignment 9

Due March 18, 2016

(73) [1, Section 4.2] Let $T\colon \mathbb{P}_3 \to \mathbb{R}, p \mapsto p(3)$, be the map that evaluates a polynomial p at $x = 3$.
 (a) Show that T is linear.
 (b) Determine the kernel and the range of T.
 (c) Is T injective, surjective, bijective?

(74) [1, Section 4.4]
 (a) Let $B = (\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix})$ be a basis of a subspace H of $\mathbb{R}^3$. Compute the coordinates $[u]_B$ for $u = \begin{bmatrix} -5 \\ 11 \\ 5 \end{bmatrix}$ in H.
 (b) Let $C = (1 + t, t + t^2, 1 + t^2)$ be a basis for $\mathbb{P}_2$. Compute the coordinates $[p]_C$ for $p = 2 + t^2$.

(75) [1, Section 4.6]
 (a) If A is a 3×4-matrix, what is the largest possible rank of A? What is the smallest possible dimension of Nul A?
 (b) If the nullspace of a 4×6-matrix B has dimension 3, what is the dimension of the row space of B?

(76) [1, Sections 4.3-4.6] True or false? Explain your answers:
 (a) Any plane in $\mathbb{R}^3$ is isomorphic to $\mathbb{R}^2$.
 (b) A basis for V is a linear independent set that is as large as possible.
 (c) If $v_1, \ldots, v_k$ are linearly independent in V, then $k \leq \dim V$.
 (d) If B is an echelon form of A, then the pivot columns of B are a basis for Col A.
 (e) The row space of A^T is equal to the column space of A.

(77) [1, Section 3.1] Compute the determinant of the matrices by cofactor expansion. Pick a row or column that yields the least amount of computation:

$$A = \begin{bmatrix} 0 & 1 & -3 \\ 5 & 4 & -4 \\ 0 & -3 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & -3 & 0 \\ 3 & 1 & 5 & 1 \\ 2 & 0 & 0 & 0 \\ 7 & 1 & -2 & 5 \end{bmatrix}.$$

(78) [1, Section 3.1] **Rule of Sarrus for the determinant of 3×3-matrices.** Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Prove that

$$\det A = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

Hint: Expand $\det A$ across the first row.

(79) [1, Section 3.1] Give two 3×3-matrices with determinat 5. (Hint: triangular matrices.)

(80) [1, Section 3.2] Compute the determinants by row reduction to echelon form:

$$A = \begin{bmatrix} 3 & 3 & -3 \\ 3 & 4 & -4 \\ 2 & -3 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{bmatrix}$$

(81) [1, Section 3.2] Consider $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

(a) How does switching the rows effect the determinant? Compare $\det A$ and $\det \begin{bmatrix} c & d \\ a & b \end{bmatrix}$.

(b) How does adding a multiple of one row to the other row effect the determinant? Compare $\det A$ and $\det \begin{bmatrix} a & b \\ c + ra & d + rb \end{bmatrix}$.

References

[1] David C. Lay. Linear Algebra and Its Applications. Addison-Wesley, 4th edition, 2012.