Math 3130 - Assignment 8

Due March 11, 2016

(64) Let U, V, W be vector spaces, and let $f\colon U \to V$ and $g\colon V \to W$ be linear mappings.
  (a) Show that the composition mapping $h\colon U \to W$, $\mathbf{u} \mapsto g(f(\mathbf{u}))$ is linear.
  (b) Does it make sense to ask whether $k\colon V \to V$, $\mathbf{u} \mapsto f(g(\mathbf{v}))$ is linear?

(65) Let U, V be vector spaces and $T\colon U \to V$ be a linear mapping. Show that $T(\mathbf{0}) = \mathbf{0}$.
  Hint: Write down $T(\mathbf{0} + \mathbf{0})$ in two different ways.

(66) Let U, V be vector spaces and $T\colon U \to V$ be a linear mapping. Show that the range $T(U)$ is a subspace of V.

(67) Let $V = \{f\colon \mathbb{R} \to \mathbb{R}\}$ be a vector space of functions. Is the set $\{\cos t, \sin t, \sin(t + \frac{\pi}{4})\}$ linearly independent?
  Hint: Use a formula for expanding $\sin(\alpha + \beta)$.

(68) Let $B = (\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix})$ and $C = (\begin{bmatrix}2\\5\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix})$ be bases of $\mathbb{R}^2$.
  (a) Find the standard matrix for $f\colon \mathbb{R}^2 \to \mathbb{R}^2$, $[\mathbf{u}]_B \mapsto \mathbf{u}$.
  (b) Find the standard matrix for $g\colon \mathbb{R}^2 \to \mathbb{R}^2$, $\mathbf{u} \mapsto [\mathbf{u}]_C$.
  (c) Find the standard matrix for $h\colon \mathbb{R}^2 \to \mathbb{R}^2$, $[\mathbf{u}]_B \mapsto [\mathbf{u}]_C$. Hint: $h(\mathbf{x}) = g(f(\mathbf{x}))$.

(69) Determine the standard matrix for the reflection T of $\mathbb{R}^2$ at the line $3x + y = 0$ as follows:
  (a) Find a basis B of $\mathbb{R}^2$ whose vectors are easy to reflect.
  (b) Give the standard matrix for the reflection relative to the coordinate system determined by B.
  (c) Use the change of coordinate matrix P_B to compute the standard matrix with respect to the standard basis $E = (e_1, e_2)$.

(70) Let $\mathbf{b}_1 = \begin{bmatrix}1\\2\\-1\end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix}1\\1\\3\end{bmatrix}$, $\mathbf{b}_3 = \begin{bmatrix}1\\2.5\\-5\end{bmatrix}$.
  (a) Find vectors $\mathbf{u}_1, \ldots, \mathbf{u}_k$ such that $(\mathbf{b}_1, \mathbf{b}_2, \mathbf{u}_1, \ldots, \mathbf{u}_k)$ is a basis for $\mathbb{R}^3$.
  (b) Find vectors $\mathbf{v}_1, \ldots, \mathbf{v}_\ell$ such that $(\mathbf{b}_3, \mathbf{v}_1, \ldots, \mathbf{v}_\ell)$ is a basis for $\mathbb{R}^3$.
  Prove that your choices for (a) and (b) form a basis.

(71) Let

$$A = \begin{bmatrix} -5 & 8 & 0 & -17 & -2 \\ 3 & -5 & 1 & 5 & 1 \\ 11 & -19 & 7 & 1 & 3 \\ 7 & -13 & 5 & -3 & 1 \end{bmatrix}.$$

  Find bases and dimensions for $\operatorname{Nul} A$, $\operatorname{Col} A$, and $\operatorname{Row} A$, respectively.

(72) A 177×35 matrix A has 19 pivots. Find $\dim \operatorname{Nul} A$, $\dim \operatorname{Col} A$, $\dim \operatorname{Row} A$, and $\operatorname{rank} A$.