Math 3130 - Assignment 7

Due March 4, 2016

(55) Let $B = (b_1, \dots, b_n)$ be a basis for a vector space V and consider the coordinate mapping $V \to \mathbb{R}^n,\ x \mapsto [x]_B$.
   (a) Show that $[c \cdot x]_B = c[x]_B$ for all $x \in V, c \in \mathbb{R}$.
   (b) Show that the coordinate mapping is onto $\mathbb{R}^n$.

(56) Let $\mathbb{P}_2$ be the vector space of polynomials of degree at most 2, and let $D : \mathbb{P}_2 \to \mathbb{P}_2, f \mapsto f'$, be the linear map that computes the derivative of a polynomial.
   (a) Determine kernel and range of D.
   (b) Is D injective, surjective, bijective?

(57) Which of the following are bases of $\mathbb{R}^3$? Why or why not?

$$A = (\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}), B = (\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}, \begin{bmatrix}0\\-1\\4\end{bmatrix}), C = (\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix})$$

(58) Give a basis for Nul A and a basis for Col A for

$$A = \begin{bmatrix} 0 & 2 & 0 & 3 \\ 1 & -4 & -1 & 0 \\ -2 & 6 & 2 & -3 \end{bmatrix}$$

(59) Give 2 different bases for

$$H = \text{Span}\{\begin{bmatrix}1\\1\\2\end{bmatrix}, \begin{bmatrix}3\\-1\\0\end{bmatrix}, \begin{bmatrix}-1\\3\\4\end{bmatrix}\}$$

(60) Show that $\cos t, \cos 2t$ are linearly independent in the vector space of real valued functions.

(61) Consider the vector space of functions $V = \text{Span}\{\cos t, 2\cos t, \cos 2t, 3\cos 2t\}$. Give a basis for V.

(62) Let $B = (\begin{bmatrix}1\\-2\end{bmatrix}, \begin{bmatrix}-3\\4\end{bmatrix})$ be a basis of $\mathbb{R}^2$.
   (a) Give the change of coordinates matrix P_B from B to the standard basis $E = (e_1, e_2)$.
   (b) Find vectors $u, v \in \mathbb{R}^2$ with $[u]_B = \begin{bmatrix}0\\1\end{bmatrix}, [v]_B = \begin{bmatrix}3\\2\end{bmatrix}$.
   (c) Compute the coordinates relative to B of $w = \begin{bmatrix}-2\\4\end{bmatrix}$ and $x = \begin{bmatrix}1\\0\end{bmatrix}$.

(63) Let $B = (1, t, t^2)$ and $C = (1, 1+t, 1+t+t^2)$ be bases of $\mathbb{P}_2$.
   (a) Determine the polynomials p, q with $[p]_B = \begin{bmatrix}3\\0\\-2\end{bmatrix}$ and $[q]_C = \begin{bmatrix}3\\0\\-2\end{bmatrix}$.
   (b) Compute $[r]_B$ and $[r]_C$ for $r = 3 + 2t + t^2$.