Math 3130 - Assignment 6

Due February 26, 2016

(46) Let $v_1, \dots, v_n$ be vectors in a vector space V. Show that $H := \text{Span}\{v_1, \dots, v_n\}$ is a subspace of V.

(47) Let A be an $m \times n$ matrix. Prove that the Nullspace of A is a subspace of $\mathbb{R}^n$.

(48) Let $M_{2\times2}$ be the set of all 2×2 matrices. Let $+$ be the sum of matrices and $\cdot$ be the multiplication of a matrix by a scalar.
- (a) Show that $M_{2\times2}$ forms a vector space.
- (b) Let H be the set of invertible 2×2 matrices. Show that H is not a subspace of $M_{2\times2}$.

(49) Show that $V := \{\begin{bmatrix} a \\ b \end{bmatrix} \mid a, b \in \mathbb{R},\ a \geq 0\}$ is no subspace of $\mathbb{R}^2$.

(50)
- (a) Which of the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}$ are in $\text{Nul}\, A$?
- (b) Which of the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}$ are in $\text{Col}\, A$?

$$\mathbf{u} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -1 \\ 0 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 2 & 4 \\ 2 & -4 & 1 & 0 \\ -3 & 6 & 2 & 7 \end{bmatrix}$$

(51) Let A be the matrix from (50).
- (a) Solve $A\mathbf{x} = \mathbf{0}$ and give the solution in parametrized vector form.
- (b) Find vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^4$ such that

$$\text{Nul}\, A = \{r\mathbf{u} + s\mathbf{v} \mid r, s \in \mathbb{R}\}.$$

(52) Let $V := \{f : \mathbb{R} \to \mathbb{R}\}$ be the vector space of functions on $\mathbb{R}$.
- (a) Is $\{f : \mathbb{R} \to \mathbb{R} \mid f(0) = 1\}$ a subspace of V?
- (b) Is $\{f : \mathbb{R} \to \mathbb{R} \mid f(1) = 0\}$ a subspace of V?
- (c) Is $\{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is continuous}\}$ a subspace of V?

(53) We want to rotate the triangle with corners $A = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ for $270°$ (counterclockwise) around B.
- (a) Use homogenous coordinates to find matrices that describe the translation T by $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and the rotation R about B.
- (b) Compute the rotated points A, B, C.
- (c) Draw the triangle and its rotated version.

(54) Are the vectors $\mathbf{u} = 1$, $\mathbf{v} = t$, $\mathbf{w} = t^2$ in the vector space $V := \{f : \mathbb{R} \to \mathbb{R}\}$ linearly independent?