# Math 3130 - Assignment 4

Due February 12, 2016

(28) Let $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T\colon \mathbb{R}^3 \to \mathbb{R}^2$, $\mathbf{x} \mapsto \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \mathbf{x}$.

  (a) Find the solution set of $T(\mathbf{x}) = \mathbf{b}$ in parametric vector form.

  (b) Give 2 vectors in $\mathbb{R}^3$ which are mapped to $\mathbf{b}$ by $T$, and give 2 vectors in $\mathbb{R}^3$ which are not mapped to $\mathbf{b}$ by $T$.

(29) Let $T$ be as in (28) and let $A$ be the standard matrix of $T$.

  (a) Do the colums of $A$ span $\mathbb{R}^2$?

  (b) Are the columns of $A$ linearly independent?

  (c) Is $T$ injective? Is $T$ surjective? Is $T$ bijective?

(30) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear map such that

$$T(\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, T(\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

  (a) Express $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ as linear combination of $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$.

  (b) Use the linearity of $T$ to find $T(\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix})$ and $T(\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix})$.

(31) Let

$$\mathbf{a}_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \ \mathbf{a}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \ \mathbf{a}_3 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \ \mathbf{a}_4 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

  (a) Find $x_1, x_2, x_3, x_4 \in \mathbb{R}$ such that $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 + x_4\mathbf{a}_4 = \mathbf{b}$. Verify your solution.

  (b) Let $: \mathbb{R}^3 \to \mathbb{R}$ be a linear mapping such that $T(\mathbf{a}_1) = 10$, $T(\mathbf{a}_2) = 6$, $T(\mathbf{a}_3) = 8$, $T(\mathbf{a}_4) = 26$. Compute $T(\mathbf{b})$.

(32) Let 0 be the 0-matrix with size $2 \times 2$. Find $2 \times 2$ matrices $A \neq 0$ and $B \neq 0$ such that $AB = 0$.

(33) (a) If possible, invert the following matrices:

$$A = \begin{bmatrix} 2 & -3 \\ 4 & -9 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 2 \\ 4 & -3 \end{bmatrix}$$

  (b) For which $a \in \mathbb{R}$ can the following matrix be inverted? Compute the inverse of $C$.

$$C = \begin{bmatrix} a-2 & 1 \\ 4 & a \end{bmatrix}$$

(34) Show that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

if $ad - bc \neq 0$.

(35) If possible, invert the following matrix:

$$D = \begin{bmatrix} -3 & 2 & 4 \\ 0 & 1 & -2 \\ 1 & -3 & 4 \end{bmatrix}$$

(36) If possible, invert the following matrix:

$$E = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ -1 & 2 & -1 \end{bmatrix}$$