Math 3130 - Assignment 2

Due February 5, 2016

(19) Show that the following maps are not linear by giving concrete vectors for which the defining properties of linear maps are not satisfied.

(a) $g : \mathbb{R}^2 \to \mathbb{R}^2, \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} xy \\ y \end{bmatrix}$

(b) $h : \mathbb{R}^2 \to \mathbb{R}^2, \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} |x| + |y| \\ 2x \end{bmatrix}$

(20) [1, Section 1.8, Ex 24] An *affine transformation* $T : \mathbb{R}^n \to \mathbb{R}^m$ has the form $T(\mathbf{x}) = A\mathbf{x} + b$ with A an $m \times n$-matrix and $b \in \mathbb{R}^m$. Show that T is not a linear transformation if $b \neq 0$.

(21) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear map such that

$$T(\begin{bmatrix} 1 \\ 2 \end{bmatrix}) = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}, T(\begin{bmatrix} 3 \\ 2 \end{bmatrix}) = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$$

(a) Use the linearity of T to find $T(\begin{bmatrix} 1 \\ 0 \end{bmatrix})$ and $T(\begin{bmatrix} 0 \\ 1 \end{bmatrix})$.

(b) Determine $T(\begin{bmatrix} x \\ y \end{bmatrix})$ for arbitrary $x, y \in \mathbb{R}$.

(22) Give the standard matrices for the following linear transformations:

(a) $T : \mathbb{R}^2 \to \mathbb{R}^3, \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 2x + y \\ x \\ -x + y \end{bmatrix}$;

(b) the function S on $\mathbb{R}^2$ that scales all vectors to half their length.

(23) Give the standard matrix for the linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$ that rotates points (about the origin) by 60° counterclockwise and then reflects them on the x-axis.

(24) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection at the line $2x + 3y = 0$. Note that T is linear.

(a) What is the reflection of the normal vector $\mathbf{a} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ of the line? What is the reflection of the vector $\mathbf{b} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, which is on this line? Make a drawing if necessary.

(b) Write the unit vectors $\mathbf{e}_1, \mathbf{e}_2$ as linear combinations of $\mathbf{a}$ and $\mathbf{b}$.

(c) Use the linearity of T to find the reflection of the unit vectors $T(\mathbf{e}_1), T(\mathbf{e}_2)$ from $T(\mathbf{a}), T(\mathbf{b})$.

(d) Give the standard matrix for T.

(25) Is

$$T : \mathbb{R}^3 \to \mathbb{R}^2, x \mapsto \begin{bmatrix} 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix} \cdot x$$

injective, surjective, bijective?

(26) [1, cf. Section 1.9, Ex 23/24] True or False? Correct the false statements to make them true.

(a) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is completely determined by the images of the unit vectors in $\mathbb{R}^n$.
(b) $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto $\mathbb{R}^m$ if every vector $\in \mathbb{R}^n$ is mapped onto some vector in $\mathbb{R}^m$.
(c) $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if every vector $\in \mathbb{R}^n$ is mapped onto a unique vector in $\mathbb{R}^m$.
(d) A linear map $T : \mathbb{R}^3 \to \mathbb{R}^2$ cannot be one-to-one.

(27) Compute if possible

$$A + 3B, B \cdot A, A \cdot B, A \cdot C, C \cdot A$$

for the matrices

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix}, C = \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 1 & 3 \end{bmatrix}.$$

If an expression is undefined, explain why.

References

[1] David C. Lay. Linear Algebra and Its Applications. Addison-Wesley, 4th edition, 2012.