

## Math 3130 - Assignment 2

Due January 29, 2016

- (10) [1, Section 1.4, Ex 17] Does the row echelon form of  $A$  contain any rows that are all 0? Does the equation  $A\mathbf{x} = \mathbf{b}$  have a solution for each  $\mathbf{b} \in \mathbb{R}^4$ ?

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & -1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$$

- (11) [1, Section 1.4, Ex 31] Let  $A$  be a  $3 \times 2$  matrix. Explain why the equation  $A\mathbf{x} = \mathbf{b}$  cannot have a solution for all  $\mathbf{b} \in \mathbb{R}^n$ .
- (12) Let  $\mathbf{u} \in \mathbb{R}^n$  be a vector and let  $c, d \in \mathbb{R}$  be scalars. Show that

$$(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}.$$

- (13) [1, cf. Section 1.5, Ex 17] Let

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 8 \\ -16 \\ 12 \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Solve the equations  $A\mathbf{x} = \mathbf{b}$  and  $A\mathbf{x} = \mathbf{0}$ . Express both solution sets in parametric vector form. Give a geometric description of the solution sets.

- (14) [1, cf. Section 1.5, Ex 11] Let

$$A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Solve the equations  $A\mathbf{x} = \mathbf{b}$  and  $A\mathbf{x} = \mathbf{0}$ . Express both solution sets in parametric vector form.

- (15) [1, Section 1.5, Ex 31] Let  $A$  be a  $3 \times 2$  matrix with 2 pivot positions (i.e., its echelon form has 2 non-zero rows).

(a) Does the equation  $A\mathbf{x} = \mathbf{0}$  have a nontrivial solution?

(b) Does the equation  $A\mathbf{x} = \mathbf{b}$  have a solution for every possible  $\mathbf{b} \in \mathbb{R}^3$ ?

Explain your answers!

- (16) [1, Section 1.7, Ex 9] Let

$$\mathbf{u} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}.$$

(a) For which values of  $h$  is  $\mathbf{w}$  in  $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ ?

(b) For which values of  $h$  is  $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$  linearly dependent?

- (17) [1, cf. Section 1.7, Ex 21] Mark each statement True or False, and justify each answer.

(a) The columns of a matrix  $A$  are linearly independent if  $\mathbf{x} = \mathbf{0}$  is a solution of  $A\mathbf{x} = \mathbf{0}$ .

(b) If  $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$  is linearly dependent, then each vector is a linear combination of the other two vectors.

(c) The columns of any  $4 \times 5$  matrix are linearly dependent.

- (d) If  $\mathbf{u}$  and  $\mathbf{v}$  are linearly independent, and if  $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$  is linearly dependent, then  $\mathbf{w}$  is in the span of  $\mathbf{u}, \mathbf{v}$ .
- (18) Show the following Theorem in 2 steps: Suppose  $A\mathbf{x} = \mathbf{b}$  has a solution  $\mathbf{p}$ . Then the set of all solutions of  $A\mathbf{x} = \mathbf{b}$  is

$$\mathbf{p} + \text{Nul}A = \{\mathbf{p} + \mathbf{v} \mid \mathbf{v} \in \text{Nul}A\}.$$

Suppose  $A\mathbf{x} = \mathbf{b}$  has a solution  $\mathbf{p}$ .

- (a) Show that if  $\mathbf{v}$  is in  $\text{Nul}A$ , then  $\mathbf{p} + \mathbf{v}$  is also a solution for  $A\mathbf{x} = \mathbf{b}$ .
- (b) Show that if  $\mathbf{q}$  is a solution for  $A\mathbf{x} = \mathbf{b}$ , then  $\mathbf{q} - \mathbf{p}$  is in  $\text{Nul}A$ .

#### REFERENCES

- [1] David C. Lay. Linear Algebra and Its Applications. Addison-Wesley, 4th edition, 2012.