Math 2135 Spring 2019 - Review for Finals

Numbers refer to sections in Treil, Linear algebra done wrong.

1. **Systems of linear equations.**
   (1) coefficient and augmented matrix (2.3)
   (2) solving a linear system by row reduction, pivot columns, free variables, solution in parametrized vector form (2.3)
   (3) solutions of homogenous vs. inhomogenous systems (2.6)
2. **Fields.**
   (1) axioms of fields, examples $\mathbb{R}, \mathbb{Q}, \mathbb{C}, \mathbb{Z}_2, \mathbb{Z}_3, \ldots$, properties of fields (Halmos 1.1)
3. **Vector spaces.**
   (1) operations and their properties for vector spaces over arbirtrary fields (1.1)
   (2) examples of vector spaces: tuples, functions, polynomials P_n
   (3) subspaces: definition and examples (span, null space) (1.7)
4. **Basis of vector spaces.**
   (1) dimension (2.5), Basis Theorem (cf. 2.3.1)
   (2) reduce a spanning set to a basis (1.2.1), extend a linear independent set to a basis (2.5.1)
   (3) bases and dimension for column space, row space, null space of a matrix (2.7.1-2.7.3)
   (4) coordinates with respect to a basis B (1.2)
   (5) change of coordinate matrix $[id]_{B,C}$ for bases B and C (2.8.3)
   (6) orthogonal basis, coordinates via dot product (5.1.1, 5.2), orthogonal projection (5.3)
5. **Matrices.**
   (1) matrix product and composition of linear maps (1.5)
   (2) inverse matrices and their properties, Invertible Matrix Theorem (1.6)
   (3) inverse matrix via row reduction (2.4)
   (4) formula for inverse of 2×2-matrix
   (5) determinant via cofactor expansion and via row reduction (3.1)
   (6) eigenvalues and eigenvectors of matrices, characteristic polynomials (4.1)
   (7) diagonalizing matrices, powers of matrices (4.2)

6. **Linear maps.**
   (1) a linear map is determined by its images on a basis (1.3.3)
   (2) matrix $[f]_{B,C}$ of a linear map f with respect to bases B, C, standard matrix $[f]_{E,E}$ (for standard basis E) (2.8.2)
   (3) matrix for rotation, reflection in $\mathbb{R}^2$ and $\mathbb{R}^3$ (cf. 1.5.2)
   (4) injective, surjective, bijective linear maps and connections with kernel, range (cf. 1.7)
   (5) isomorphism between vector spaces, n-dimensional vector space is isomorphic to F^n (1.6.3)