Multiplication of matrix by vector

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Definition. For $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \in \mathbb{R}^{m\times n}$ and $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$,

define

$A \cdot \mathbf{x} :=$

Lemma. Let $A \in \mathbb{R}^{m\times n}$, $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, and $c \in \mathbb{R}$. Then

(1) $A \cdot (\mathbf{x} + \mathbf{y}) = A \cdot \mathbf{x} + A \cdot \mathbf{y}$
(2) $A \cdot (c\mathbf{x}) = c(A \cdot \mathbf{x})$

Proof. For (1) consider

$A \cdot (\mathbf{x} + \mathbf{y}) =$