# Math 2135 - Assignment 8

Due March 15, 2019

(1) Compute the inverse if possible:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 2 & 4 \\ 0 & 1 & -2 \\ 1 & -3 & 4 \end{bmatrix}$$

(2) (a) Give examples of square matrices $A, B$ with $AB \neq BA$.
 (b) Let $A, B \in F^{n\times n}$ be invertible. Show $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$.

(3) Let $A$ be an **upper triangular matrix**, that is,

$$A = \begin{bmatrix} a_{11} & \dots & \dots & a_{1n} \\ 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & a_{nn} \end{bmatrix}$$

with zeros below the diagonal. Show
 (a) $A$ is invertible iff there are no zeros in the diagonal of $A$.
 (b) If $A^{-1}$ exists, it is an upper triangular matrix as well.
 Hint: When row reducing $[A, I_n]$ to $[I_n, A^{-1}]$, what happens to the $n$ columns on the right?

(4) Assume that $f\colon F^n \to F^n,\ x \mapsto A \cdot x$, is bijective. Show that $A \in F^{n\times n}$ is invertible.
 Give a formula for the inverse function $f^{-1}$.
 Hint: Use that $f$ is onto $F^n$, the Basis Theorem and the Invertible Matrix Theorem.

(5) Let $P_3$ the vector space of polynomials of degree $\leq 3$ over $\mathbb{R}$ with basis $B = (1, x, x^2, x^3)$.
 (a) Find the matrix $[d]_{B,B}$ for the derivation map $d\colon P_3 \to P_3, p \to p'$.
 (b) Use $[d]_{B,B}$ to compute $[p']_B$ and $p'$ for the polynomial $p$ with $[p]_B = (-3, 2, 0, 1)$.

(6) Let $B = (\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix})$ and $C = (\begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix})$ be bases of $\mathbb{R}^2$.
 (a) Find the standard matrix for $f\colon \mathbb{R}^2 \to \mathbb{R}^2,\ [u]_B \mapsto u$.
 (b) Find the standard matrix for $g\colon \mathbb{R}^2 \to \mathbb{R}^2,\ u \mapsto [u]_C$.
 (c) Find the standard matrix for $h\colon \mathbb{R}^2 \to \mathbb{R}^2,\ [u]_B \mapsto [u]_C$.
 Hint: $h(x) = g(f(x))$.

(7) Determine the standard matrix for the reflection $t$ of $\mathbb{R}^2$ at the line $3x + y = 0$ as follows:
 (a) Find a basis $B$ of $\mathbb{R}^2$ whose vectors are easy to reflect.
 (b) Give the matrix $[t]_{B,B}$ for the reflection with respect to the coordinate system determined by $B$.
 (c) Use the change of coordinate matrix to compute the standard matrix $[t]_{E,E}$ with respect to the standard basis $E = (e_1, e_2)$.

(8) (a) Determine the standard matrix $A$ for the rotation $r$ of $\mathbb{R}^3$ around the $z$-axis through the angle $\pi/3$ counterclockwise.
 Hint: Use the matrix for the rotation around the origin in $\mathbb{R}^2$ for the $xy$-plane.

(b) Consider the rotation $s$ of $\mathbb{R}^3$ around the line spanned by $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ through the angle $\pi/3$ counterclockwise. Find a basis of $\mathbb{R}^3$ for which the matrix $[s]_{B,B}$ is equal to $A$ from (a).

(c) Give the standard matrix $[s]_{E,E}$ for the standard basis $E$ (You do not need to actually multiply and invert the involved matrices; the product formula is enough).