Math 2135 - Assignment 5

Due February 22, 2019

Problems 1-5 are revising material for the first midterm on February 20. So you should try to solve them before Wednesday!

(1) Show that $\cos x, \cos 2x$ are linearly independent in the vector space of functions $\mathbb{R}^{\mathbb{R}}$ over $\mathbb{R}$.

(2) Consider the vector space of functions $V = \operatorname{Span}(\cos x, 2\cos x, \cos 2x, 3\cos 2x)$ over $\mathbb{R}$. Give a basis for V.

(3) Explain whether the following are true or false (give counter examples if possible):

(a) Vectors v_1, v_2, v_3 are linearly dependent if v_2 is a linear combination of v_1, v_3.

(b) A subset $\{v\}$ of a vector space is linearly dependent iff $v = 0$.

(c) Two vectors in $\mathbb{R}^3$ cannot span all of $\mathbb{R}^3$.

(d) There exist four vectors in $\mathbb{R}^3$ that are linearly independent.

(4) Let $v_1, \ldots, v_n$ be linearly independent in a vector space V. Show that no vector in $\operatorname{Span}(v_1, \ldots, v_n)$ can be expressed by two different linear combinations.

Hint: Use contraposition. Assume some vector u can be written as linear combination with distinct lists of coefficients $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$. Show that $v_1, \ldots, v_n$ is linearly dependent.

(5) Prove the following or give a counter example:

(a) In general, the column space and the row space of a matrix A are not the same.

(b) If a matrix B is a row echelon form of A, then the pivot columns of B are a basis for $\operatorname{Col} A$.

(6) (a) Find vectors $u_1, \ldots, u_k$ such that $\left(\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, u_1, \ldots, u_k\right)$ is a basis of $\mathbb{R}^3$.

(b) Find vectors $v_1, \ldots, v_k$ such that $\left(\begin{bmatrix} 2 \\ 1 \\ -5 \end{bmatrix}, v_1, \ldots, v_k\right)$ is a basis of $\mathbb{R}^3$.

Check that your choices form bases.

(7) Let $B = \left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \end{bmatrix}\right)$ be a basis of $\mathbb{R}^2$.

(a) Find vectors $u, v \in \mathbb{R}^2$ with $[u]_B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, [v]_B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

(b) Compute the coordinates relative to B of $w = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$.

(8) Let $B = (1, x, x^2)$ and $C = (1, 1+x, 1+x+x^2)$ be bases of P_2 (polynomial functions of degree ≤ 2).

(a) Determine the polynomials p, q with $[p]_B = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$ and $[q]_C = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$.

(b) Compute $[r]_B$ and $[r]_C$ for $r = 3 + 2x + x^2$.