Math 2135 - Assignment 4

Due February 15, 2019

(1) Explain why the following are not subspaces of $\mathbb{R}^2$ over the field $\mathbb{R}$. Give explicit counter examples for subspace properties that are not satisfied.

(a) $U = \{\begin{bmatrix} x \\ y \end{bmatrix} \mid x, y \in \mathbb{R}, x \geq 0\}$

(b) $V = \mathbb{Q}^2$

(c) $W = \{\begin{bmatrix} x \\ y \end{bmatrix} \mid x, y \in \mathbb{R}, |x| = |y|\}$

(2) Which of the following are subspaces of the vector space $\mathbb{R}^{\mathbb{R}} = \{f \colon \mathbb{R} \to \mathbb{R}\}$ of all functions over $\mathbb{R}$? Check all subspace properties or give one that is not satisfied.

(a) $\{f \colon \mathbb{R} \to \mathbb{R} \mid f(0) = 1\}$

(b) $\{f \colon \mathbb{R} \to \mathbb{R} \mid f(1) = 0\}$

(c) $\{f \colon \mathbb{R} \to \mathbb{R} \mid f \text{ is continuous}\}$

(3) Explain whether the following are true or false (give counter examples if possible):

(a) Every vector space is a subspace of itself.

(b) Each plane in $\mathbb{R}^3$ is a subspace.

(c) Let U be a subspace of a vector space V. Any linear combination of vectors of U is also in V.

(d) Let $v_1, \dots, v_n$ be in a vector space V. Then $\mathrm{Span}(v_1, \dots, v_n)$ is the smallest subspace of V containing $v_1, \dots, v_n$.

(4) Which of the following sets of vectors is linearly independent?

(a) $\begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -11 \\ 0 \end{bmatrix}$

(5) Are the functions $1, x, x^2$ in the vector space $\mathbb{R}^{\mathbb{R}} = \{f \colon \mathbb{R} \to \mathbb{R}\}$ linearly independent?

Hint: Consider a linear combinaion of these functions and evaluate it at some specific points $x = 0, \dots$ to get several equations to solve for the coefficients.

(6) Which of the following are bases of $\mathbb{R}^3$? Why or why not?

$$A = (\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}), B = (\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}), C = (\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix})$$

(7) Give a basis for $\mathrm{Nul}\, A$ and a basis for $\mathrm{Col}\, A$ for

$$A = \begin{bmatrix} 0 & 2 & 0 & 3 \\ 1 & -4 & -1 & 0 \\ -2 & 6 & 2 & -3 \end{bmatrix}$$

(8) Give 2 different bases for

$$U = \mathrm{Span}(\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix})$$