Math 2135 - Assignment 2

Due February 8, 2019

(1,2) Let V be a vector space over the field F with zero vector $\mathbf{0}$, let $\mathbf{v} \in V$ and $c \in F$. Prove that

$$c\mathbf{v} = \mathbf{0} \text{ iff } c = 0 \text{ or } \mathbf{v} = \mathbf{0}.$$

Show the implication $\Leftarrow$ as exercise (1) and $\Rightarrow$ as exercise (2). State the axioms that you are using.

Hint for (2): Suppose $c\mathbf{v} = \mathbf{0}$ and $c \neq 0$. How can you deduce that $\mathbf{v} = \mathbf{0}$?

(3) Is $\mathbf{b}$ a linear combination of the vectors $\mathbf{a}_1, \mathbf{a}_2$?

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -2 \\ -3 \\ 3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

(4) For which values of a is $\mathbf{b}$ in the plane spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$?

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} a \\ -3 \\ -5 \end{bmatrix}$$

(5) (a) Find vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k \in \mathbb{R}^3$ that span the plane in $\mathbb{R}^3$ with equation $x-2y+3z = 0$. How many do you need?
Hint: Write down a parametrized solution for the equation.

(b) Continuing problem 1 of assignment 2: Find vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k \in \mathbb{R}^4$ such that $\operatorname{Nul} A = \operatorname{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_k)$ for

$$A = \begin{bmatrix} 0 & 0 & 2 & 4 \\ 2 & -4 & 1 & 0 \\ -3 & 6 & 2 & 7 \end{bmatrix}$$

(6) Are the following true or false? Explain your answers.

(a) For every $A \in \mathbb{R}^{2\times 3}$ with 2 pivots, $Ax = 0$ has a nontrivial solution.

(b) For every $A \in \mathbb{R}^{2\times 3}$ with 2 pivots and every $\mathbf{b} \in \mathbb{R}^2$, $Ax = \mathbf{b}$ is consistent.

(c) The vector $3\mathbf{v}_1$ is a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2$.

(d) For $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^3$, $\operatorname{Span}(\mathbf{v}_1, \mathbf{v}_2)$ is always a plane through the origin.

(7) Let $\mathbf{v}_1, \ldots, \mathbf{v}_n$ be vectors in a vector space V over some field F. Complete the proof from class that $U := \operatorname{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_n)$ is a subspace of V.

(8) Let $A \in F^{m\times n}$ for a field F. Prove that the nullspace of A, $\operatorname{Nul} A$, is a subspace of F^n.