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(10) 1 / 1 / (a)

Quiz 6 Review:

$$1) [\text{id}]_{B,E} = \begin{bmatrix} [b_1]_E & [b_2]_E \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$[\text{id}]_{E,B} = [\text{id}]_{B,E}^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^{-1}$$

Recall: matrix of  $f$  with respect to bases  $B, C$

$$[f]_{B,C} = \begin{bmatrix} [f(b_1)]_C & [f(b_2)]_C \\ \vdots & \vdots \end{bmatrix}$$

$$\text{Then } [f(v)]_C = [f]_{B,C} \cdot [v]_B$$

$$2) [f]_{B,E} = \begin{bmatrix} f(b_1) & f(b_2) \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$$

$$f(b) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} =$$

End of Review

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Recall:  $V, W$  are isomorphic (Greek: iso = same; morphus = form) if there exists a bijective linear map  $f: V \rightarrow W$

Ex. For  $B$  a basis of  $V$ ,  $\dim V = n$ ,

$V \rightarrow F^n$ ,  $v \mapsto [v]_B$ , is an isomorphism

Ex. Vector space of  $2 \times 2$  matrices over  $\mathbb{R}$  has basis  $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

isomorphic to  $\mathbb{R}^4$

COR. Every  $n$ -dimensional over  $F$  is isomorphic to  $F^n$ .

Proof: By Coordinate map.  $\square$

Ex.  $\mathbb{Q}^2, \mathbb{R}^2$  are not isomorphic since they are vector spaces over different fields.

### 3.1 Determinants

Recall:  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$  s.t.  
 $ad-bc \neq 0$

$ad-bc$  is the Determinant of the  $2 \times 2$  matrix

\* Def. The Determinant of a  $2 \times 2$  matrix  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is

$$\det A = ad - bc$$

$$\det [a] = a \quad (1 \times 1 \text{ matrix})$$

Def. The determinant of an  $n \times n$  matrix for  $n \geq 2$ ,

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}$$

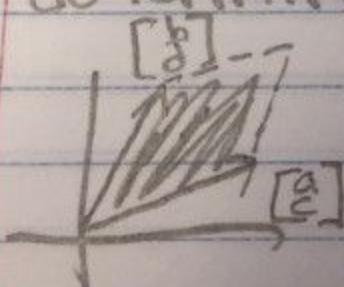
is the alternating sum

$$(*) \det A = a_{11} \cdot \det A_{11} - a_{12} \cdot \det A_{12} + \dots + (-1)^{n+1} a_{1n} \det A_{1n}$$
$$= \sum_{j=1}^n (-1)^{1+j} a_{1j} \cdot \det A_{1j}$$

Where  $A_{ij}$  is the  $(n-1) \times (n-1)$  matrix

obtained from  $A$  by deleting row  $i$  and column  $j$

Idea:  $\det A$  is the volume of the object determined by the columns of  $A$



(\*) is called the Cofactor expansion  
of  $\det A$  across the first row of  $A$

Ex.  $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 4 \\ -2 & 5 & -3 \end{bmatrix}$

$$A_{11} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 5 & -3 \end{bmatrix} \quad \det A_{11} = 0 \cdot \det [-3] \\ - 4 \det [5]$$
$$= 0(-3) - 4 \cdot 5 = -20$$

$$A_{12} = \begin{bmatrix} 3 & -1 \\ 2 & -3 \end{bmatrix} \quad \det A_{12} =$$