

(10/28/19)

Homework 8 problem 8:

a) rotation in  $\mathbb{R}^3$  around  $z$  axis:

$$A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

-standard matrix for rotation according to " $\theta$ ".

b) rotation around  $\text{span}([\underbrace{\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}}_{b_3}])$

$$n(b_3) = b_3$$

We want vectors  $b_1, b_2$  perpendicular to  $b_3$ , but also  $\perp$  to each other and both of length 1.

$b_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$  is orthogonal to  $b_3$ .

$b_2 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$   $\perp$  to  $b_3$  but not to  $b_1$ .

example:  $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \perp \begin{bmatrix} y \\ -x \\ 0 \end{bmatrix}$

$b_1 = \frac{1}{\sqrt{10}} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$  is of length 1

$b_2 = \frac{1}{\sqrt{10}} \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$  is of length 1

Recall: to convert  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  to length 1, multiply by  $\frac{1}{\sqrt{x^2+y^2+z^2}}$

For  $B = (b_1, b_2, b_3)$

$$[r]_{B,B} = A$$

c)  $[r]_{E,E} = [\text{id}]_{B,E} \cdot [v]_{B,B} \cdot [\text{id}]_{E,B}$

change in coordinates  
matrices

Recall:  $[\text{id}]_{B,E} = \begin{bmatrix} 1 & 0 & 0 \\ b_1 & b_2 & b_3 \end{bmatrix}$

$$[\text{id}]_{E,B} = [\text{id}]_{B,E}^{-1}$$

Recall:  $[r(v)]_B = [\text{id}]_{B,B} \cdot [v]_B$

$$[v]_B = [\text{id}]_{E,B} \cdot [v]_E$$

End of Review

Q: When is a linear map Bijective? (2.7)

Recall: 1)  $f: A \rightarrow B$  is 1 to 1 ~~if~~ (injective)  
if  $\forall x, y \in A \quad f(x) = f(y) \Rightarrow x = y$

2)  $f: A \rightarrow B$  is subjective (onto) if  $f(A) = B$ .

3)  $f: A \rightarrow B$  is Bijective if 1) and  
2) holds

Def: Let  $f: V \rightarrow W$  be linear

The range (image) of  $f$  is  $f(V) := \{f(x) \mid x \in V\}$

The kernel of  $f$  is

$$\ker f := \{x \in V \mid f(x) = \vec{0}_w\}$$

Ex. For  $f: F^n \rightarrow F^m$ ;  $x \mapsto A \cdot x$

$$f(F^n) = \text{Col } A$$

$$\ker f = \text{Null } A$$