Math 2135 - Assignment 13

Due December 6, 2019

(1) (a) Give 3 vectors of length 1 in $\mathbb{R}^3$ that are orthogonal to $u = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

(b) Which of the following are orthonormal sets?

$$A = \{\begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}, \begin{bmatrix} 0.8 \\ -0.6 \end{bmatrix}\}, \qquad B = \{\frac{1}{3}\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \frac{1}{\sqrt{18}}\begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}\}$$

(2) (a) Let W be the subspace of $\mathbb{R}^3$ with orthonormal basis $B = (\frac{1}{3}\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix})$. Compute the coordinates $[x]_B$ for $x = \begin{bmatrix} 7 \\ 4 \\ 4 \end{bmatrix}$ in W using dot products.

(b) Find the closest point to $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in W. What is the distance from y to W?

(3) Let $W = \operatorname{Span}(\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix})$ be a subspace of $\mathbb{R}^3$. Compute a basis for the orthogonal complement $W^\perp$ of W.

(4) Let W be a subspace of $\mathbb{R}^n$. Show that its orthogonal complement

$$W^\perp := \{x \in \mathbb{R}^n \mid x \text{ is orthogonal to all } w \in W\}$$

is also a subspace of $\mathbb{R}^n$.

(5) Continuation of the previous problem: Show that

(a) $W \cap W^\perp = 0$

(b) $\dim W + \dim W^\perp = n$

Hint: Let $w_1, \ldots, w_k$ be a basis of W. Use that $x \in W^\perp$ iff x is orthogonal to $w_1, \ldots, w_k$.

(6) Use the Gram-Schmidt algorithm to find orthonormal bases for the following subspaces:

$$U = \operatorname{Span}(\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix}), \qquad W = \operatorname{Span}(\begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 4 \end{bmatrix})$$

(7) Use the Gram-Schmidt process to transform the vectors in an orthonormal set.

$$x_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, x_2 = \begin{bmatrix} -1 \\ 1 \\ 3 \\ -3 \end{bmatrix}, x_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(8) True or false. Explain your answers.

(a) Every orthogonal set is also orthonormal.

(b) Not every orthonormal set in $\mathbb{R}^n$ is linearly independent.

(c) For each x and each subspace W, the vector $x - \operatorname{proj}_W(x)$ is orthogonal to W.

(d) Multiplying the vectors in an orthogonal basis by non-zero scalars yields again an orthogonal basis.