

Math 2135 - Assignment 8

Due October 25, 2019

- (1) Compute the inverse if possible:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 2 & 4 \\ 0 & 1 & -2 \\ 1 & -3 & 4 \end{bmatrix}$$

- (2) (a) Give examples of square matrices A, B with $AB \neq BA$.
(b) Let $A, B \in F^{n \times n}$ be invertible. Show $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$.
(3) Let A be an **upper triangular matrix**, that is,

$$A = \begin{bmatrix} a_{11} & \cdots & \cdots & a_{1n} \\ 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & a_{nn} \end{bmatrix}$$

with zeros below the diagonal. Show

- (a) A is invertible iff there are no zeros in the diagonal of A .
(b) If A^{-1} exists, it is an upper triangular matrix as well.
Hint: When row reducing $[A, I_n]$ to $[I_n, A^{-1}]$, what happens to the n columns on the right?
- (4) Assume that $f: F^n \rightarrow F^n$, $x \mapsto A \cdot x$, is bijective. Show that $A \in F^{n \times n}$ is invertible. Give a formula for the inverse function f^{-1} .
Hint: Use that f is onto F^n , the Basis Theorem and the Invertible Matrix Theorem.
- (5) Let P_3 the vector space of polynomials of degree ≤ 3 over \mathbb{R} with basis $B = (1, x, x^2, x^3)$.
(a) Find the matrix $[d]_{B,B}$ for the derivation map $d: P_3 \rightarrow P_3, p \rightarrow p'$.
(b) Use $[d]_{B,B}$ to compute $[p']_B$ and p' for the polynomial p with $[p]_B = (-3, 2, 0, 1)$.
- (6) Let $B = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$ and $C = \left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right)$ be bases of \mathbb{R}^2 , let E be the standard basis of \mathbb{R}^2 .
(a) Find the standard matrix for $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $[u]_B \mapsto [u]_E$ (change of coordinates map).
(b) Find the standard matrix for $g: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $[u]_E \mapsto [u]_C$.
(c) Find the standard matrix for $h: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $[u]_B \mapsto [u]_C$.
Hint: $h(x) = g(f(x))$.
- (7) Determine the standard matrix for the reflection t of \mathbb{R}^2 at the line $3x + y = 0$ as follows:
(a) Find a basis B of \mathbb{R}^2 whose vectors are easy to reflect.
(b) Give the matrix $[t]_{B,B}$ for the reflection with respect to the coordinate system determined by B .
(c) Use the change of coordinate matrix to compute the standard matrix $[t]_{E,E}$ with respect to the standard basis $E = (e_1, e_2)$.
- (8) (a) Determine the standard matrix A for the rotation r of \mathbb{R}^3 around the z -axis through the angle $\pi/3$ counterclockwise.

Hint: Use the matrix for the rotation around the origin in \mathbb{R}^2 for the xy -plane.

- (b) Consider the rotation s of \mathbb{R}^3 around the line spanned by $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ through the angle $\pi/3$ counterclockwise. Find a basis of \mathbb{R}^3 for which the matrix $[s]_{B,B}$ is equal to A from (a).
- (c) Give the standard matrix $[s]_{E,E}$ for the standard basis E (You do not need to actually multiply and invert the involved matrices; the product formula is enough).