Math 2135 - Assignment 3

Due September 20, 2019

(1) Use row reduction to solve the following linear system over $\mathbb{Z}_2$ (for brevity we write $0, 1$ for the classes $[0], [1]$, respectively):

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \cdot x = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

(2) Let F be a field. Show that $F^{2\times 2}$ forms a vector space with $+$ the componentwise sum of matrices and $\cdot$ the componentwise multiplication of a matrix by a scalar. In particular, determine what is the zero vector and what is the additive inverse of a matrix in $F^{2\times 2}$.

(3,4) Let V be a vector space over the field F with zero vector 0, let $\mathrm{v} \in V$ and $c \in F$. Prove that

$$c\mathrm{v} = 0 \text{ iff } c = 0 \text{ (the scalar 0) or } \mathrm{v} = 0 \text{ (the zero vector).}$$

Show the implication $\Leftarrow$ as exercise (3) and $\Rightarrow$ as exercise (4). State the axioms that you are using.

Hint for (4): Suppose $c\mathrm{v} = 0$ and $c \neq 0$. How can you deduce that $\mathrm{v} = 0$?

(5) Is b a linear combination of the vectors $\mathrm{a}_1, \mathrm{a}_2$?

$$\mathrm{a}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \mathrm{a}_2 = \begin{bmatrix} -2 \\ -3 \\ 3 \end{bmatrix}, \mathrm{b} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

(6) For which values of a is b in the plane spanned by v_1 and v_2?

$$\mathrm{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \mathrm{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}, \mathrm{b} = \begin{bmatrix} a \\ -3 \\ -5 \end{bmatrix}$$

(7) (a) Find vectors $\mathrm{v}_1, \dots, \mathrm{v}_k \in \mathbb{R}^3$ that span the plane in $\mathbb{R}^3$ with equation $x - 2y + 3z = 0$. How many do you need?
Hint: Write down a parametrized solution for the equation.

(b) Continuing problem 3 of assignment 2: Find vectors $\mathrm{v}_1, \dots, \mathrm{v}_k \in \mathbb{R}^4$ such that $\operatorname{Nul} A = \operatorname{Span}(\mathrm{v}_1, \dots, \mathrm{v}_k)$ for

$$A = \begin{bmatrix} 0 & 0 & 2 & 4 \\ 2 & -4 & 1 & 0 \\ -3 & 6 & 2 & 7 \end{bmatrix}$$

(8) Are the following true or false? Explain your answers.

(a) For every $A \in \mathbb{R}^{2\times 3}$ with 2 pivots, $Ax = 0$ has a nontrivial solution.

(b) For every $A \in \mathbb{R}^{2\times 3}$ with 2 pivots and every $\mathrm{b} \in \mathbb{R}^2$, $Ax = \mathrm{b}$ is consistent.

(c) The vector $3\mathrm{v}_1$ is a linear combination of the vectors $\mathrm{v}_1, \mathrm{v}_2$.

(d) For $\mathrm{v}_1, \mathrm{v}_2 \in \mathbb{R}^3$, $\operatorname{Span}(\mathrm{v}_1, \mathrm{v}_2)$ is always a plane through the origin.