Math 2135 - Assignment 1

Due September 06, 2019

Solve all systems of linear equations by row reduction (Gaussian elimination) and give the solutions in parametric vector form.

(1) Compute if possible:

(a) $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} =$ (b) $\begin{bmatrix} 2 & -1 \\ 3 & 1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} =$

(2) Let $A \in \mathbb{R}^{m\times n}$ be a matrix, let $x, y \in \mathbb{R}^n$ be vectors, let $c \in \mathbb{R}$ be a scalar. Show that

(a) $A(x + y) = Ax + Ay$,

(b) $A(cx) = c(Ax)$.

Hint: Write $A, x, \ldots$ in general form $A = \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \ldots & a_{mn} \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \ldots$ and use the definitions of the product, ...

(3) Do the following 4 planes intersect in a point? Which?

$$\begin{aligned} x + 5y + 3z &= 16 \\ 2x + 10y + 8z &= 34 \\ 4x + 20y + 15z &= 67 \\ x + 6y + 5z &= 21 \end{aligned}$$

(4) Add an equation of a line to the equation

$$2x + 3y = 4$$

such that the resulting system has (a) no solution, (b) exactly one solution, (c) infinitely many solutions.

(5) Solve the system of linear equations with augmented matrix

$$\begin{bmatrix} 0 & 0 & 2 & 4 \\ 2 & -4 & 1 & 0 \\ -3 & 6 & 2 & 7 \end{bmatrix}$$

(6) Solve the system of linear equations with augmented matrix

$$\begin{bmatrix} 2 & 2 & 1 & 8 & 2 \\ 2 & 0 & 0 & 8 & 2 \\ 2 & 6 & 3 & 8 & 1 \end{bmatrix}$$

(7) Let

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 8 \\ -16 \\ 12 \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Solve $Ax = b$ and $Ax = \mathbf{0}$. Express both solution sets in parametric vector form. Give a geometric description of the solution sets.

(8) Let

$$A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Solve the equations $Ax = b$ and $Ax = \mathbf{0}$. Express both solution sets in parametric vector form.