Math 2135 - Assignment 8

Due October 26, 2018

(1) (a) Find vectors $u_1, \ldots, u_k$ such that $\left(\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, u_1, \ldots, u_k\right)$ is a basis of $\mathbb{R}^3$.

(b) Find vectors $v_1, \ldots, v_k$ such that $\left(\begin{bmatrix} 2 \\ 1 \\ -5 \end{bmatrix}, v_1, \ldots, v_k\right)$ is a basis of $\mathbb{R}^3$.

Check that your choices form bases.

(2) (a) An 8×5-matrix A has 4 pivot columns. Find $\dim \operatorname{Nul} A$, $\dim \operatorname{Col} A$.

(b) If B is a 3×4-matrix, what is the largest possible dimension of $\operatorname{Col} B$? What is the smallest possible dimension of $\operatorname{Nul} B$?

(c) If the nullspace of a 4×6-matrix C has dimension 3, what is $\dim \operatorname{Col} C$?

(3) Let V be a vector space that is spanned by a finite set of vectors $v_1, \ldots, v_n$. Show that V is finite dimensional.

Hint: Why does V have a finite basis?

(4) Prove the following or give a counter example:

(a) A basis B for a vector space V is a linear independent list of vectors in V that is as large as possible.

(b) If $k > \dim V$, then any set of k vectors in V are linearly dependent.

(5) The **row space** $\operatorname{Row} A$ of a matrix A is the span of the rows of A.

Show that if matrices A and B are row equivalent, then $\operatorname{Row} A = \operatorname{Row} B$.

Hint: Check for each type of elementary row operation, that it does not change the row space.

(6) Show that if B is a matrix in row echelon form, then its non-zero rows are a basis for $\operatorname{Row} B$.

(7) Problem (5) and (6) together prove the following:

Theorem. Let A, B be row equivalent matrices and B in row echelon form. Then the non-zero rows of B form a basis for $\operatorname{Row} A$.

(a) Use this to find a basis for the row space of

$$A = \begin{bmatrix} 0 & 2 & -3 \\ 1 & 0 & 1 \\ 2 & 2 & -1 \end{bmatrix}.$$

(b) What can you say about the relation between $\dim \operatorname{Col} A$ and $\dim \operatorname{Row} A$ for arbitrary matrices A?

(8) Prove the following or give a counter example:

(a) In general, the column space and the row space of a matrix A are not the same.

(b) If a matrix B is in row echelon form and row equivalent to A, then the pivot columns of B are a basis for $\operatorname{Col} A$.